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Symmetric Remote Single-Qubit State Preparation via Positive Operator-Valued Measurement

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Abstract I present a tripartite scheme for realizing remote single-qubit state preparation in either distant ministrant's place. In this scheme, to fulfill the remote preparation, the preparer performs a two-qubit projective measurement in a properly chosen bases. Then after the two ministrants' collaboration, one of them successfully realizes the remote single-qubit state preparation in a probabilistic manner by executing a proper positive operator-valued measurement (POVM). The success probability and classical communication cost in the scheme are also calculated. Furthermore, I also explore its applications to two special ensembles of states in detail. The extensive investigations show that the remote preparation can be achieved with higher probability provided that the prepared state belongs to the two special ensembles. Finally, I sketch the generalization of the tripartite scheme to a multiparty case.

Keywords Remote state preparation · Two-qubit projective measurement · Positive operator-valued measurement · Local unitary operation

1 Introduction

Applying quantum entanglement, people have put forward various quantum protocols, such as quantum teleportation [1], quantum dense coding [2], quantum secret sharing [3], remote state preparation [4], and so on [5–25]. Quantum teleportation was first proposed by Bennett et al. [1] in 1993. It is a method for interchanging quantum resources between different places via quantum entanglement. In 2000, another interesting novel method to interchange quantum resources was formally proposed by Lo [4]. It also utilizes a prior shared entanglement and some classical communication to treat pure quantum states. Conventionally, this new communication protocol is termed as remote state preparation (RSP) and viewed as "teleportation of a known state". In RSP the prepared state is assumed to be completely

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known by the sender. In contrast, the teleported state is not required to be known by the sender in quantum teleportation. Moreover, due to the prior knowledge about the original state, to some extend the classical communication and entanglement cost can be reduced in RSP. For example, Pati [5] has shown that for a qubit chosen from equatorial or polar great circles on a Bloch sphere, RSP procedure requires only 1 forward classical bit, exactly half that of quantum teleportation. However, for general states RSP procedure requires as much communication cost as quantum teleportation. The detailed trade-off between the classical communication cost and the required entanglement in RSP procedure was studied distinctly in the protocol proposed by Bennett et al. [6].

In the last decade, RSP has attracted much attention [26–49]. In these protocols, different entangled states are employed for the quantum channel. In terms of entanglements in quantum channels, these RSP schemes can be classified into two types. One uses maximally entangled states [40–44] while another utilizes non-maximally entangled states [45–49]. In the latter case, one or more auxiliary qubits need to be incorporated and entangled with the original qubits. After this, a proper measurements on qubits including the ancillas should be executed such that the original-qubit state is collapsed to one of the eligible states. Subsequently, the prepared state is retrieved from the eligible state by performing an appropriate unitary operations which correspond to the measurement outcomes. Note that, the so-called proper measurements are projective measurements in the latter type of existing RSP schemes [45–49]. As a matter of fact, there lies another type of measurement named positive operator-valued measurement (POVM) [50, 51]. Recently it has already attracted much attention and been employed in various quantum information processing [51–55]. In view of that I am led to ask if it is possible to *implement remote single-qubit state preparation using POVM* when the employed quantum channel consists of *two non-maximally entangled two-qubit states*. In this contribution I show that it is indeed possible to construct such protocol. Furthermore, the success probability and classical communication cost in the scheme are also calculated.

This paper is organized as follows. In Sect. 2, I exhibit a tripartite RSP scheme via POVM in detail and explore its applications to two special ensembles of states. Then I sketch the generalization of the tripartite scheme to a multiparty case. Finally, I give brief summaries in Sect. 3.

2 The RSP Scheme and the Exploration of Its Applications

Now let me present the tripartite symmetric scheme. Suppose Alice is the state preparer, Bob and Charlie are her two remote ministrants. Alice, Bob and Charlie share in priori two non-maximally entangled two-qubit states

$$|\psi\rangle_{12} = a|00\rangle_{12} + b|11\rangle_{12}, \qquad |\psi'\rangle_{34} = c|00\rangle_{34} + d|11\rangle_{34}, \tag{1}$$

where the real coefficients a, b, c and d satisfy $a^2 + b^2 = 1$, $c^2 + d^2 = 1$, and $|a| \geq |b|$, $|c| \geq |d|$. Qubit pair (1, 3) belongs to Alice while qubits 2, 4 to Bob and Charlie, respectively. Alice wants to prepare remotely a state with the two ministrants' help. The prepared state is normalized and written as $|V\rangle = \alpha|0\rangle + \beta|1\rangle$, where α is real and β is complex. Alice knows it exactly while Bob and Charlie do not. Owing to the channel symmetry for two ministrants, either can construct the state $|V\rangle$ with another's assistance. Specifically, Charlie can construct it with Bob's help and vice versa. Without loss of generality, hereafter suppose

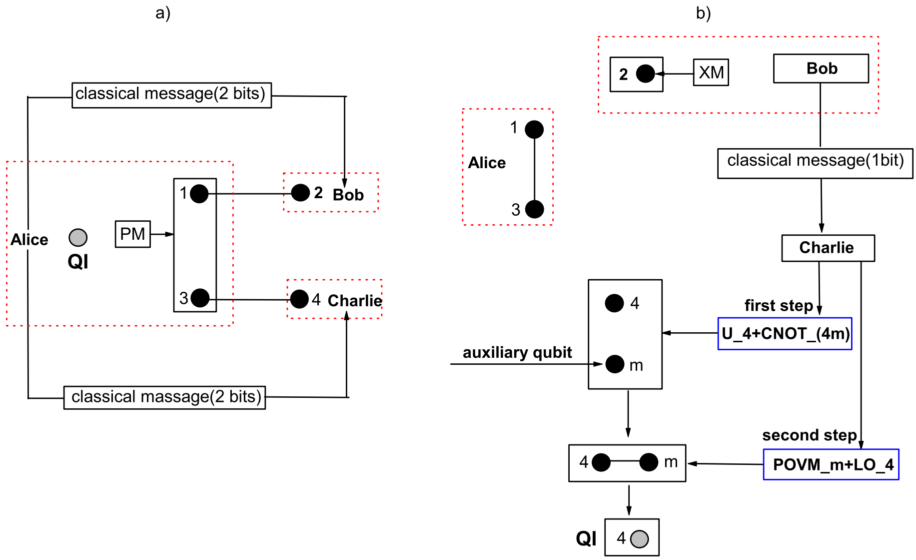

Fig. 1 Symmetric tripartite remote single-qubit state preparation. The *gray dot* denotes the quantum information (QI, a quantum state) to be prepared; XM denotes a single-qubit projective measurement (PM) in the X bases; U denotes the unitary operation on qubit 4 before the controlled-not (CNOT) gate operation; LO denotes a local appropriate unitary operation on qubit 4 after Charlie's POVM operation. See text for more details

Charlie is assigned to construct the prepared state. To achieve her goal, Alice performs a projective measurement on her qubits (1, 3). The measurement basis chosen by Alice is a set of mutually orthogonal basis vectors $\{|\lambda_1\rangle, |\lambda_2\rangle, |\lambda_3\rangle, |\lambda_4\rangle\}$, which are related to the usual computation bases $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ as

$$\begin{aligned} |\lambda_1\rangle &= (\alpha|00\rangle + \beta|11\rangle), \qquad |\lambda_2\rangle = (\beta^*|00\rangle - \alpha|11\rangle), \\ |\lambda_3\rangle &= (\alpha|01\rangle + \beta|10\rangle), \qquad |\lambda_4\rangle = (\beta^*|01\rangle - \alpha|10\rangle). \end{aligned} \tag{2}$$

Then the combined state of the quantum channel can be expressed as

$$\begin{aligned} |\psi\rangle_{12} \otimes |\psi'\rangle_{34} &= ac|0000\rangle_{1234} + ad|0011\rangle_{1234} + bc|1100\rangle_{1234} + bd|1111\rangle_{1234} \\ &= |\lambda_1\rangle_{13}(\alpha ac|00\rangle_{24} + \beta^* bd|11\rangle_{24}) \\ &\quad + |\lambda_2\rangle_{13}(\beta ac|00\rangle_{24} - \alpha bd|11\rangle_{24}) \\ &\quad + |\lambda_3\rangle_{13}(\alpha ad|01\rangle_{24} + \beta^* bc|10\rangle_{24}) \\ &\quad + |\lambda_4\rangle_{13}(\beta ad|01\rangle_{24} - \alpha bc|10\rangle_{24}). \end{aligned} \tag{3}$$

After Alice's measurement, she broadcasts her measurement result (shown in Fig. 1a) in terms of their prior agreements, i.e., 00 correspond to $|\lambda_1\rangle$, 01 to $|\lambda_2\rangle$, 10 to $|\lambda_3\rangle$ and 11 to $|\lambda_4\rangle$. According to the above equation, one can see that Alice's measurement result should be one of the four states defined in (2). Without loss of generality, suppose Alice measures $|\lambda_2\rangle_{13}$. In this case, the collapsed state of qubits (2, 4) is $\beta ac|00\rangle_{24} - \alpha bd|11\rangle_{24}$. As mentioned before, Charlie is assigned to construct the prepared state with Bob's help. Hence, after Alice's publication on her measurement result, she lets Bob measure his qubit

2 in the X bases. This measurement is a single-qubit projective measurement in a set of two mutually orthogonal basis vectors

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \qquad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \tag{4}$$

In the bases, the collapsed state of qubits (2, 4) can be expressed as

$$\beta ac|00\rangle_{24} - \alpha bd|11\rangle_{24} = \frac{1}{\sqrt{2}}[|+\rangle_2(\beta ac|0\rangle_4 - \alpha bd|1\rangle_4) + |-\rangle_2(\beta ac|0\rangle_4 + \alpha bd|1\rangle_4). \tag{5}$$

According to this equation, if Bob's measurement result is $|+\rangle_2$, the state of qubit 4 in Charlie's place will collapse to $\frac{1}{\sqrt{2}}(\beta ac|0\rangle_4 - \alpha bd|1\rangle_4)$. Otherwise, it collapses to $\frac{1}{\sqrt{2}}(\beta ac|0\rangle_4 + \alpha bd|1\rangle_4)$. At this stage, in order to construct the original state, Charlie cooperates with Bob to get his help. Provided Bob agrees to cooperate with Charlie, he will communicate his measurement result to Charlie over a public channel. To be specifical, Bob sends Charlie classical message 0 (1) to show his measurement result $|+\rangle_2$ ($|-\rangle_2$). Without loss of generality, suppose Bob measures $|+\rangle_2$. Upon receiving Bob's message, Charlie retrieves the prepared state on his qubit 4 as follows (shown in Fig. 1b). First Charlie performs a local unitary operation $U_1 = |1\rangle\langle 0| - |0\rangle\langle 1|$ on his qubit 4, which transforms the collapsed state of qubit 4 into

$$U_1 \frac{1}{\sqrt{2}}(\beta ac|0\rangle_4 - \alpha bd|1\rangle_4) = \frac{1}{\sqrt{2}}(\alpha bd|0\rangle_4 + \beta ac|1\rangle_4). \tag{6}$$

Subsequently, Charlie introduces an auxiliary qubit m in the state $|0\rangle$. After the incorporation, Charlie performs a controlled-not (CNOT) gate operation on the auxiliary qubit m with qubit 4 as the controlled qubit while the auxiliary qubit m as the target one. The CNOT operation C_{4m} converts the state of the qubits 4 and m to

$$\begin{aligned}
|T\rangle_{4m} &= C_{4m}\left[\frac{1}{\sqrt{2}}(\alpha bd|0\rangle_4 + \beta ac|1\rangle_4)|0\rangle_m\right] \\
&= \frac{1}{\sqrt{2}}(\alpha bd|00\rangle_{4m} + \beta ac|11\rangle_{4m}) \\
&= \frac{1}{2\sqrt{2}}(|V\rangle_4 \otimes |R\rangle_m + |V'\rangle_4 \otimes |R'\rangle_m),
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
|V\rangle_4 &= \alpha|0\rangle_4 + \beta|1\rangle_4, \qquad |V'\rangle_4 = \alpha|0\rangle_4 - \beta|1\rangle_4, \\
|R\rangle_m &= bd|0\rangle_m + ac|1\rangle_m, \qquad |R'\rangle_m = bd|0\rangle_m - ac|1\rangle_m.
\end{aligned}$$

From (7), one can see that, $|V\rangle_4$ is exactly the prepared state. Readily, the prepared state can be further retrieved from $|V'\rangle_4$. Moreover, note that Charlie can get the state $|V\rangle_4$ or $|V'\rangle_4$ provided that the states $|R\rangle_m$ and $|R'\rangle_m$ are distinguished via an appropriate measurement. Unfortunately, the states $|R\rangle_m$ and $|R'\rangle_m$ are not orthogonal in general. As a consequence, it is impossible to precisely and deterministically distinguish these two states. Nevertheless, the discrimination can be achieved in a probabilistic manner by making an optimal POVM [50, 51] on the ancillary qubit m as follows:

$$P_1 = \frac{1}{\omega}|M_1\rangle\langle M_1|, \qquad P_2 = \frac{1}{\omega}|M_2\rangle\langle M_2|, \qquad P_3 = I - \frac{1}{\omega}\sum_{i=1}^{2}|M_i\rangle\langle M_i|, \tag{8}$$

where

$$|M_1\rangle = \frac{1}{\sqrt{\xi}}\left(\frac{1}{bd}|0\rangle + \frac{1}{ac}|1\rangle\right)_m, \qquad |M_2\rangle = \frac{1}{\sqrt{\xi}}\left(\frac{1}{bd}|0\rangle - \frac{1}{ac}|1\rangle\right)_m,$$
$$\xi = \frac{1}{(bd)^2} + \frac{1}{(ac)^2} = \frac{1 - d^2 - b^2 + 2b^2d^2}{b^2(1-b^2)d^2(1-d^2)},$$

I is an identity operator, and ω is a coefficient relating to ac and bd, which should be able to assure P_3 to be a positive operator. To exactly determine ω, I would like to rewrite the three operators P_1, P_2 and P_3 in the matrix form

$$P_1 = \frac{1}{\omega\xi}\begin{pmatrix} \frac{1}{(bd)^2} & \frac{1}{abcd} \\ \frac{1}{abcd} & \frac{1}{(ac)^2} \end{pmatrix}, \qquad P_2 = \frac{1}{\omega\xi}\begin{pmatrix} \frac{1}{(bd)^2} & -\frac{1}{abcd} \\ -\frac{1}{abcd} & \frac{1}{(ac)^2} \end{pmatrix}, \qquad P_3 = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix},$$

where

$$A = 1 - \frac{2}{\omega\xi(bd)^2}, \qquad B = 1 - \frac{2}{\omega\xi(ac)^2}.$$

Obviously, the coefficient ω should be chosen such that all the diagonal elements of P_3 are nonnegative. So ω should be an appropriate value within the scope $1 \le \omega \le 2$, as is strongly relative to ac and bd.

After performing the above POVM operation on his auxiliary qubit m, Charlie can definitively get $|R\rangle_m$ or $|R'\rangle_m$ with equal probability

$$p \equiv {}_{4m}\langle T|P_1|T\rangle_{4m} = {}_{4m}\langle T|P_2|T\rangle_{4m} = \frac{1}{2\omega\xi}.$$

Alternatively, in terms of the POVM value Charlie can positively conclude the state of qubit m. Accordingly, Charlie knows exactly the state of his qubit 4 with probability $2p$ after his POVM. As a consequence, Charlie can construct the prepared state $|V\rangle$ by performing a local appropriate unitary operation. Explicitly, if Charlie knows the state of qubit m is $|R\rangle_m$ or $|R'\rangle_m$, it means the state of his qubit 4 is $|V\rangle_4$ or $|V'\rangle_4$, then to construct the prepared state on qubit 4, he needs only to perform the local unitary operation $I_4 = |0\rangle_4\langle 0| + |1\rangle_4\langle 1|$ or $\sigma_4^z = |0\rangle_4\langle 0| - |1\rangle_4\langle 1|$, respectively. However, with the probability $1 - 2 \times p = 1 - \frac{1}{\omega\xi}$ Charlie may get P_3's value. In this case, Charlie can not infer which state the qubit m is in. It means the RSP protocol fails. So far I have depicted the case that Bob measures $|+\rangle_2$. As mentioned before Bob may get $|-\rangle_2$. In the latter case, the RSP process is trivially similar to that in the former one. Alternatively, Charlie can also construct the prepared state on his qubit 4 with the same success probability. Both possible cases are listed in Table 1 and here I do not depict it anymore. Thus, the total success probability in the case that Alice measures $|\lambda_2\rangle_{13}$ is

$$4p = \frac{2}{\omega\xi} = \frac{2}{\omega} \times \frac{(b^2 - b^4)(d^2 - d^4)}{1 - b^2 - d^2 + 2b^2d^2}, \tag{9}$$

Table 1 Alice's measurement results (AM) and classical bits (C_A), Bob's measurement results (BM) and classical bits (C_B), the collapsed states (CS) of qubit 4. The local unitary operations (U) before performing the CNOT gate operation. The elements of POVM (P_i) and the succedent appropriate local unitary operations (LO) need to be performed on qubit 4. See text for more details

AM	C_A	BM	C_B	CS	U	P_i	LO
$\|\lambda_2\rangle_{13}$	01	$\|+\rangle_2$	0	$\beta ac\|0\rangle_4-\alpha bd\|1\rangle_4$	$\|1\rangle\langle 0\|-\|0\rangle\langle 1\|$	P_1	I_4
$\|\lambda_2\rangle_{13}$	01	$\|+\rangle_2$	0	$\beta ac\|0\rangle_4-\alpha bd\|1\rangle_4$	$\|1\rangle\langle 0\|-\|0\rangle\langle 1\|$	P_2	σ_4^z
$\|\lambda_2\rangle_{13}$	01	$\|-\rangle_2$	1	$\beta ac\|0\rangle_4+\alpha bd\|1\rangle_4$	$\|1\rangle\langle 0\|+\|0\rangle\langle 1\|$	P_1	I_4
$\|\lambda_2\rangle_{13}$	01	$\|-\rangle_2$	1	$\beta ac\|0\rangle_4+\alpha bd\|1\rangle_4$	$\|1\rangle\langle 0\|+\|0\rangle\langle 1\|$	P_2	σ_4^z

and the required amount of classical communication is

$$4\times\frac{1}{2\omega\xi}\log_2[2\omega\xi]=\frac{2}{\omega}\times\frac{(b^2-b^4)(d^2-d^4)}{1-b^2-d^2+2b^2d^2}\log_2\left[2\omega\times\frac{1-b^2-d^2+2b^2d^2}{(b^2-b^4)(d^2-d^4)}\right]\text{ bits.}$$

If Alice's measurement result is $|\lambda_4\rangle_{13}$, then the classical bits 11 are broadcasted. According to (3), one can see the joint state of qubits (2, 4) will collapse to $\beta ad|01\rangle_{24}-\alpha bc|10\rangle_{24}$. Similarly, in order to realize the preparation, Bob is asked to measure his qubit 2 in the X bases. Note that in the bases, the joint state of qubits (2, 4) can be rewritten as

$$\beta ad|01\rangle_{24}-\alpha bc|10\rangle_{24}=\frac{1}{\sqrt{2}}[|+\rangle_2(\beta ad|1\rangle_4-\alpha bc|0\rangle_4)+|-\rangle_2(\beta ad|1\rangle_4+\alpha bc|0\rangle_4)]. \tag{10}$$

After Bob's single-qubit projective measurement, he communicates his measurement result to Charlie according to the same agreement proposed above. Upon receiving Alice and Bob's classical messages, Charlie can conclude the collapsed state of his qubit 4. Specifically, if Bob measures $|+\rangle_2$, the collapsed state of qubit 4 in Charlie's place is $\frac{1}{\sqrt{2}}(\beta ad|1\rangle_4-\alpha bc|0\rangle_4)$. Otherwise, Charlie knows his qubit 4 is in the state $\frac{1}{\sqrt{2}}(\beta ad|1\rangle_4+\alpha bc|0\rangle_4)$. Due to similarity, now still suppose Bob conclusively measures $|+\rangle_2$. In this case, Charlie can also retrieve the prepared state on his qubit 4 from the above collapsed state as follows. First Charlie performs a local unitary operation $U_2=|0\rangle\langle 0|-|1\rangle\langle 1|$ on his qubit 4, which transforms the collapsed state into

$$U_2\frac{1}{\sqrt{2}}(\beta ad|1\rangle_4-\alpha bc|0\rangle_4)=\frac{1}{\sqrt{2}}(\alpha bc|0\rangle_4+\beta ad|1\rangle_4). \tag{11}$$

Then Charlie introduces an auxiliary qubit m in the state $|0\rangle$ and performs a CNOT gate operation on the auxiliary qubit m with qubit 4 as the controlled qubit while the auxiliary qubit m as the target one. The CNOT operation C_{4m} converts the state of the qubits 4 and m to

$$\begin{aligned}|Q\rangle_{4m}&=C_{4m}\left[\frac{1}{\sqrt{2}}(\alpha bc|0\rangle_4+\beta ad|1\rangle_4)|0\rangle_m\right]=\frac{1}{\sqrt{2}}(\alpha bc|00\rangle_{4m}+\beta ad|11\rangle_{4m})\\&=\frac{1}{2\sqrt{2}}(|V\rangle_4\otimes|H\rangle_m+|V'\rangle_4\otimes|H'\rangle_m),\end{aligned} \tag{12}$$

where

$$|H\rangle_m=bc|0\rangle_m+ad|1\rangle_m,\qquad |H'\rangle_m=bc|0\rangle_m-ad|1\rangle_m.$$

From (12), one can see that, Charlie can get the state $|V\rangle_4$ or $|V'\rangle_4$ provided that the states $|H\rangle_m$ and $|H'\rangle_m$ are distinguished via an appropriate measurement. Similarly, in order to discriminate the non-orthogonal states $|H\rangle_m$ and $|H'\rangle_m$, Charlie adopts to perform an optimal POVM [50, 51] on the auxiliary qubit m. The POVM takes the form as

$$P_1' = \frac{1}{\omega}|M_1'\rangle\langle M_1'|, \qquad P_2' = \frac{1}{\omega}|M_2'\rangle\langle M_2'|, \qquad P_3' = I - \frac{1}{\omega}\sum_{i=1}^{2}|M_i'\rangle\langle M_i'|, \tag{13}$$

where

$$|M_1'\rangle = \frac{1}{\sqrt{\xi'}}\left(\frac{1}{bc}|0\rangle + \frac{1}{ad}|1\rangle\right)_m, \qquad |M_2'\rangle = \frac{1}{\sqrt{\xi'}}\left(\frac{1}{bc}|0\rangle - \frac{1}{ad}|1\rangle\right)_m,$$
$$\xi' = \frac{1}{(bc)^2} + \frac{1}{(ad)^2} = \frac{d^2+b^2-2b^2d^2}{b^2(1-b^2)d^2(1-d^2)}.$$

The above three operators P_1', P_2' and P_3' can be rewritten in the matrix form

$$P_1' = \frac{1}{\omega\xi'}\begin{pmatrix} \frac{1}{(bc)^2} & \frac{1}{abcd} \\ \frac{1}{abcd} & \frac{1}{(ad)^2}\end{pmatrix}, \qquad P_2' = \frac{1}{\omega\xi'}\begin{pmatrix} \frac{1}{(bc)^2} & -\frac{1}{abcd} \\ -\frac{1}{abcd} & \frac{1}{(ad)^2}\end{pmatrix}, \qquad P_3' = \begin{pmatrix} A' & 0 \\ 0 & B'\end{pmatrix},$$

where

$$A' = 1 - \frac{2}{\omega\xi'(bc)^2}, \qquad B' = 1 - \frac{2}{\omega\xi'(ad)^2}.$$

After the manipulation, according to the POVM value Charlie can definitively get $|H\rangle_m$ or $|H'\rangle_m$ with equal probability

$$p' \equiv {}_{4m}\langle Q|P_1'|Q\rangle_{4m} = {}_{4m}\langle Q|P_2'|Q\rangle_{4m} = \frac{1}{2\omega\xi'}.$$

Once Charlie determines $|H\rangle_m$ or $|H'\rangle_m$, it also means that the state $|V\rangle_m$ or $|V'\rangle_m$ is obtained. Further, Charlie then constructs the prepared state by performing the corresponding local unitary operation I_4 or σ_4^z on his qubit 4, respectively. As same as that proposed before, Charlie may get P_3''s value with probability $1 - 2\times p' = 1 - \frac{1}{\omega\xi}$. In this situation, he can not infer which state the qubit m is in. Consequently, the remote preparation fails. Up to now, I have depicted the case that Bob measures $|+\rangle_2$. Similarly, Bob may get $|-\rangle_2$. In this case, applying the similar analysis method, Charlie can also construct the prepared state on his qubit 4 with the probability $2p'$. Both cases are summarized in detail in the following Table 2. So the total success probability, in the case that Alice's measurement result is $|\lambda_4\rangle_{13}$, is

$$4p' = \frac{2}{\omega\xi'} = \frac{2}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2}, \tag{14}$$

and the required amount of classical communication is

$$\frac{2}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2}\log_2\left[2\omega \times \frac{d^2+b^2-2b^2d^2}{(b^2-b^4)(d^2-d^4)}\right] \text{bits}.$$

Table 2 Same as Table 1

AM	C_A	BM	C_B	CS	U	P_i'	LO
$\vert\lambda_4\rangle_{13}$	11	$\vert+\rangle_2$	0	$\beta ad\vert 1\rangle_4-\alpha bc\vert 0\rangle_4$	$\vert 0\rangle\langle 0\vert-\vert 1\rangle\langle 1\vert$	P_1'	I_4
$\vert\lambda_4\rangle_{13}$	11	$\vert+\rangle_2$	0	$\beta ad\vert 1\rangle_4-\alpha bc\vert 0\rangle_4$	$\vert 0\rangle\langle 0\vert-\vert 1\rangle\langle 1\vert$	P_2'	σ_4^z
$\vert\lambda_4\rangle_{13}$	11	$\vert-\rangle_2$	1	$\beta ad\vert 1\rangle_4+\alpha bc\vert 0\rangle_4$	$\vert 0\rangle\langle 0\vert+\vert 1\rangle\langle 1\vert$	P_1'	I_4
$\vert\lambda_4\rangle_{13}$	11	$\vert-\rangle_2$	1	$\beta ad\vert 1\rangle_4+\alpha bc\vert 0\rangle_4$	$\vert 0\rangle\langle 0\vert+\vert 1\rangle\langle 1\vert$	P_2'	σ_4^z

Above I have already shown the tripartite symmetric RSP scheme of an arbitrary single-qubit state. Now let me make some further discussions on it. As depicted previously, it is possible that Alice measures $|\lambda_1\rangle_{13}$ or $|\lambda_3\rangle_{13}$. Along this line, the joint state of qubits (2, 4) will collapse to $\alpha ac|00\rangle_{24}+\beta^* bd|11\rangle_{24}$ and $\alpha ad|01\rangle_{24}+\beta^* bc|10\rangle_{24}$, respectively. Since Charlie has no knowledge of the two coefficients α and β, neither of the above two states can be unitarily converted by Charlie into the prepared state $|V\rangle$ with Bob's help in the latter two cases. Hence the symmetric tripartite RSP fails in the latter cases. Nonetheless, it should be noted that the coefficient α is assumed to be real while β complex in the beginning. Then it is intriguing to ask whether the so-called failure turns into success provided that the prepared state belongs to some special ensembles (i.e., α and β are some special values). After my extensive investigations, I get the positive answer and find out two special ensembles.

Ensemble I: Both α and β are real. If Alice measures $|\lambda_1\rangle_{13}$, the collapsed state of qubits (2, 4) is $\alpha ac|00\rangle_{24}+\beta bd|11\rangle_{24}$. (i) Bob's measurement result is $|+\rangle_2$. In this case, in terms of Alice and Bob's classical message Charlie knows his qubit 4 is in the state $\frac{1}{\sqrt{2}}(\alpha ac|0\rangle_4+\beta bd|1\rangle_4)$. To construct the prepared state, Charlie introduces an auxiliary qubit m in the state $|0\rangle$, and then performs a CNOT gate operation with the qubit 4 as the controlled qubit while the auxiliary qubit m as the target one. The CNOT operation transforms the joint state of the two qubits 4 and m into

$$|G\rangle_{4m}=\frac{1}{2\sqrt{2}}[V\rangle_4\otimes|J\rangle_m+|V'\rangle_4\otimes|J'\rangle_m], \tag{15}$$

where

$$|J\rangle_m=ac|0\rangle_m+bd|1\rangle_m, \qquad |J'\rangle_m=ac|0\rangle_m-bd|1\rangle_m.$$

From (15), it is known if $|J\rangle_m$ and $|J'\rangle_m$ are distinguished, the state $|V\rangle$ can be constructed via an appropriate unitary operation. Very similar to that proposed above, the discrimination of the two states $|J\rangle_m$ and $|J'\rangle_m$) can be achieved in a probabilistic manner by making an optimal POVM [50, 51]. Forasmuch, Charlie then performs an optimal POVM on the auxiliary qubit m, which takes the following matrix form

$$W_1=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{(ac)^2} & \frac{1}{abcd}\\ \frac{1}{abcd} & \frac{1}{(bd)^2}\end{pmatrix}, \qquad W_2=\frac{1}{\omega\xi}\begin{pmatrix}\frac{1}{(ac)^2} & -\frac{1}{abcd}\\ -\frac{1}{abcd} & \frac{1}{(bd)^2}\end{pmatrix}, \qquad W_3=\begin{pmatrix}B & 0\\ 0 & A\end{pmatrix}.$$

After the manipulation, according to the POVM value Charlie can conclude the corresponding state of qubit m. The probability in each case is

$$p\equiv {}_{4m}\langle G|W_1|G\rangle_{4m}={}_{4m}\langle G|W_2|G\rangle_{4m}=\frac{1}{2\omega\xi}. \tag{16}$$

Table 3 Same as Table 1

AM	C_A	BM	C_B	CS	U	W_i	LO
$\|\lambda_1\rangle_{13}$	00	$\|+\rangle_2$	0	$\alpha ac\|0\rangle_4+\beta bd\|1\rangle_4$	$\|0\rangle\langle 0\|+\|1\rangle\langle 1\|$	W_1	I_4
$\|\lambda_1\rangle_{13}$	00	$\|+\rangle_2$	0	$\alpha ac\|0\rangle_4+\beta bd\|1\rangle_4$	$\|0\rangle\langle 0\|+\|1\rangle\langle 1\|$	W_2	σ_4^z
$\|\lambda_1\rangle_{13}$	00	$\|-\rangle_2$	1	$\alpha ac\|0\rangle_4-\beta bd\|1\rangle_4$	$\|0\rangle\langle 0\|-\|1\rangle\langle 1\|$	W_1	I_4
$\|\lambda_1\rangle_{13}$	00	$\|-\rangle_2$	1	$\alpha ac\|0\rangle_4-\beta bd\|1\rangle_4$	$\|0\rangle\langle 0\|-\|1\rangle\langle 1\|$	W_2	σ_4^z

Once Charlie determines $|J\rangle_m$ or $|J'\rangle_m$, it also means that the state $|V\rangle_4$ or $|V'\rangle_4$ is obtained. Further, Charlie constructs the prepared state by performing the corresponding local unitary operation I or σ_z on his qubit 4, respectively. As same as that proposed before, Charlie may get W_3's value with probability $1-2\times p=1-\frac{1}{\omega\xi}$. In this situation, he can not infer which state the qubit m is in. Consequently, the remote preparation fails. (ii) Bob gets $|-\rangle_2$. In this case, applying the similar analysis method, Charlie can also construct the prepared state on his qubit 4 with the same probability. I have summarized both cases in Table 3 and do not depict it anymore. In this way, the total success probability, in this case, is

$$4p=\frac{2}{\omega\xi}=\frac{2}{\omega}\times\frac{(b^2-b^4)(d^2-d^4)}{1-d^2-b^2+2b^2d^2}. \tag{17}$$

If Alice's measurement result is $|\lambda_3\rangle_{13}$, the collapsed state of qubits (2, 4) is $\alpha ad|01\rangle_{24}+\beta bc|10\rangle_{24}$. (i) Bob's measurement result is $|+\rangle_2$. In this case, Charlie's qubit 4 is in the state $\frac{1}{\sqrt{2}}(\alpha ad|1\rangle_4+\beta bc|0\rangle_4)$. To construct the prepared state, Charlie first performs an unitary operation $U_3=|1\rangle\langle 0|+|0\rangle\langle 1|$ on his qubit 4 which transforms the above state into $\frac{1}{\sqrt{2}}(\alpha ad|0\rangle_4+\beta bc|1\rangle_4)$. After this Charlie introduces an auxiliary qubit m in the state $|0\rangle$, and then performs a CNOT gate operation with qubit 4 as the controlled qubit while the auxiliary qubit m as the target one. The CNOT operation transforms the joint state of qubits 4, m into

$$|S\rangle_{4m}=\frac{1}{2\sqrt{2}}[V\rangle_4\otimes|K\rangle_m+|V'\rangle_4\otimes|K'\rangle_m], \tag{18}$$

where

$$|K\rangle_m=ad|0\rangle_m+bc|1\rangle_m,\qquad |K'\rangle_m=ad|0\rangle_m-bc|1\rangle_m.$$

From (18), one can see, if $|K\rangle_m$ and $|K'\rangle_m$ are distinguished, the state $|V\rangle$ can be constructed via an appropriate unitary operation. Very similar to that proposed above, to discriminate the non-orthogonal states $|K\rangle_m$ and $|K'\rangle_m$, Charlie performs an optimal POVM [50, 51] on the auxiliary qubit m, which takes the following matrix form

$$W_1'=\frac{1}{\omega\xi'}\begin{pmatrix}\frac{1}{(ad)^2} & \frac{1}{abcd}\\ \frac{1}{abcd} & \frac{1}{(bc)^2}\end{pmatrix},\qquad W_2'=\frac{1}{\omega\xi'}\begin{pmatrix}\frac{1}{(ad)^2} & -\frac{1}{abcd}\\ -\frac{1}{abcd} & \frac{1}{(bc)^2}\end{pmatrix},\qquad W_3'=\begin{pmatrix}B' & 0\\ 0 & A'\end{pmatrix},$$

After the manipulation, according to the POVM value Charlie concludes the corresponding state $|K\rangle_m$ or $|K'\rangle_m$, respectively. The probability in each case is

$$p'\equiv {}_{4m}\langle S|W_1'|S\rangle_{4m}={}_{4m}\langle S|W_2'|S\rangle_{4m}=\frac{1}{2\omega\xi'}. \tag{19}$$

Table 4 Same as Table 1

AM	C_A	BM	C_B	CS	U	W_i'	LO
$\|\lambda_3\rangle_{13}$	10	$\|+\rangle_2$	0	$\alpha ad\|1\rangle_4+\beta bc\|0\rangle_4$	$\|0\rangle\langle 0\|+\|1\rangle\langle 1\|$	W_1'	I_4
$\|\lambda_3\rangle_{13}$	10	$\|+\rangle_2$	0	$\alpha ad\|1\rangle_4+\beta bc\|0\rangle_4$	$\|0\rangle\langle 0\|+\|1\rangle\langle 1\|$	W_2'	σ_4^z
$\|\lambda_3\rangle_{13}$	10	$\|-\rangle_2$	1	$\alpha ad\|1\rangle_4-\beta bc\|0\rangle_4$	$\|0\rangle\langle 0\|-\|1\rangle\langle 1\|$	W_1'	I_4
$\|\lambda_3\rangle_{13}$	10	$\|-\rangle_2$	1	$\alpha ad\|1\rangle_4-\beta bc\|0\rangle_4$	$\|0\rangle\langle 0\|-\|1\rangle\langle 1\|$	W_2'	σ_4^z

Once Charlie determines $|K\rangle_m$ or $|K'\rangle_m$, it also means that the state $|V\rangle_4$ or $|V'\rangle_4$ is obtained. Further, Charlie constructs the prepared state by performing the corresponding local unitary operation I or σ_z on his qubit 4, respectively. Similarly, Charlie may get W_3''s value with probability $1-2\times p'=1-\frac{1}{\omega\xi'}$. In this situation, he can not infer which state the qubit m is in. Consequently, the remote preparation fails. (ii) Bob measures $|-\rangle_2$. In this case, the RSP process is trivially similar to that in the above case with the similar analysis method. Alternatively, Charlie can also construct the prepared state on his qubit 4 with the same probability $\frac{1}{\omega\xi'}$ (shown in Table 4). So the total success probability, in this case, is

$$4p'=\frac{2}{\omega\xi'}=\frac{2}{\omega}\times\frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2}. \tag{20}$$

Conclusively, for *Ensemble I* the total success probability of remote single-qubit preparation is

$$\begin{aligned}(p+p')\times 8&=\frac{4}{\omega}\times\frac{(b^2-b^4)(d^2-d^4)}{1-d^2-b^2+2b^2d^2}+\frac{4}{\omega}\times\frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2}\\&=\frac{4}{\omega}\times\frac{(b^2-b^4)(d^2-d^4)}{(1-2b^2)^2(d^2-d^4)+(b^2-b^4)}.\end{aligned} \tag{21}$$

The total amount of classical communication required in RSP is

$$\begin{aligned}S=&\frac{4}{\omega}\times\frac{(b^2-b^4)(d^2-d^4)}{1-d^2-b^2+2b^2d^2}\log_2\left[2\omega\times\frac{1-d^2-b^2+2b^2d^2}{(b^2-b^4)(d^2-d^4)}\right]\\&+\frac{4}{\omega}\times\frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2}\log_2\left[2\omega\times\frac{d^2+b^2-2b^2d^2}{(b^2-b^4)(d^2-d^4)}\right]\text{ bits.}\end{aligned} \tag{22}$$

Ensemble II: $\alpha=\frac{1}{\sqrt{2}}$ and $\beta=\frac{1}{\sqrt{2}}e^{i\theta}$ with θ being an arbitrary real parameter. In this case, the prepared state is $\frac{1}{\sqrt{2}}(|0\rangle+e^{i\theta}|1\rangle)$. While Alice measures $|\lambda_1\rangle_{13}$ or $|\lambda_3\rangle_{13}$, the collapsed state of qubits (2, 4) will be $\frac{1}{\sqrt{2}}(ac|00\rangle_{24}+e^{-i\theta}bd|11\rangle_{24})$ and $\frac{1}{\sqrt{2}}(ad|01\rangle_{24}+e^{-i\theta}bc|10\rangle_{24})$, respectively. For both cases, provided that Bob agrees to collaborate with Charlie, the state of Charlie's qubit 4 can be probabilistically transformed into $\frac{1}{\sqrt{2}}(|0\rangle_3+e^{-i\theta}|1\rangle_3)$ via the same steps (shown in Fig. 1b) described before. Evidentally, this state can be easily converted into the prepared state $\frac{1}{\sqrt{2}}(|0\rangle+e^{i\theta}|1\rangle)_3$ by performing the local unitary operation $\sigma_4^x=|1\rangle\langle 0|+|0\rangle\langle 1|$ except for an overall trivial factor $e^{-i\theta}$. It means the tripartite remote single-qubit state preparation can also be realized with the same probability and classical communication cost proposed in *Ensemble I*.

In short, the remote single-qubit state preparation can be fulfilled in one ministrant's place with another's help. The total success probability is $\frac{2}{\omega}\times\frac{(b^2-b^4)(d^2-d^4)}{1-d^2-b^2+2b^2d^2}+\frac{2}{\omega}\times$

$\frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2}$, and the required amount of classical communication is $\frac{2}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{1-d^2-b^2+2b^2d^2} \times \log_2[2\omega \times \frac{1-d^2-b^2+2b^2d^2}{(b^2-b^4)(d^2-d^4)}] + \frac{2}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2} \log_2[2\omega \times \frac{d^2+b^2-2b^2d^2}{(b^2-b^4)(d^2-d^4)}]$ bits. Nonetheless, if the prepared state belongs to the two special ensembles *I* and *II*, then the success probability can be enhanced to $\frac{4}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{1-d^2-b^2+2b^2d^2} + \frac{4}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2}$. The required amount of classical communication will be $\frac{4}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{1-d^2-b^2+2b^2d^2} \log_2[2\omega \times \frac{1-d^2-b^2+2b^2d^2}{(b^2-b^4)(d^2-d^4)}] + \frac{4}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2} \log_2[2\omega \times \frac{d^2+b^2-2b^2d^2}{(b^2-b^4)(d^2-d^4)}]$ bits. Further, if $|b| = |d| = \frac{1}{\sqrt{2}}$ and $\omega = 1$, i.e., the quantum channel consists of two Bell states and the so-called POVM becomes the common projective measurement, then the present tripartite RSP turns to be deterministic and costs 4 classical bits for any state chosen from the two special ensembles.

Now let me sketch the generalization of the tripartite scheme to a multiparty case. Suppose the prepared state is still $|V\rangle$. The quantum channel linking Alice and her n ministrants, say Bob (1st), Charlie (2nd), Dike (3rd), ..., Zarch (nth), is composed of n non-maximally entangled two-qubit states

$$\begin{aligned}
|\psi_1\rangle_{12} &= a_1|00\rangle_{12} + b_1|11\rangle_{12}, \\
|\psi_2\rangle_{34} &= a_2|00\rangle_{34} + b_2|11\rangle_{34}, \\
|\psi_3\rangle_{56} &= a_3|00\rangle_{56} + b_3|11\rangle_{56}, \\
&\vdots \\
|\psi_n\rangle_{(2n-1)2n} &= a_n|00\rangle_{(2n-1)2n} + b_n|11\rangle_{(2n-1)2n},
\end{aligned} \tag{23}$$

where the real coefficients $a_i, b_i, (i = 1, 2, 3, \ldots, n)$ satisfy $a_i^2 + b_i^2 = 1$, and $|a_i| \geq |b_i|$. Qubits $(1, 3, 5, \ldots, 2n\text{-}1)$ belong to Alice, while qubit 2 to Bob(1st), qubit 4 to Charlie(2nd), qubit 6 to Dike(3rd), ..., qubit $2n$ to Zarch(nth). To prepare the state $|V\rangle$ in a remote ministrant's place, Alice performs a n-qubit projective measurement on her qubits $(1, 3, 5, \ldots, 2n-1)$ and broadcasts the measurement result via a classical channel. Due to symmetry, anyone of the n ministrants can construct the prepared state with others' assistance. Without loss of generality, still suppose Charlie is designed to construct the prepared state. Then the other $(n-1)$ ministrants are asked to measure their respective qubits in the X bases and tell Charlie their respective measurement results. Upon receiving Alice and the other $n-1$ ministrants' classical message, Charlie retrieves the prepared single-qubit state $|V\rangle$ in a probabilistic manner via the same analysis method proposed earlier. Intuitively, the total success probability in the multiparty case is $\frac{2}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{1-d^2-b^2+2b^2d^2} + \frac{2}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2}$. While the prepared state belongs to the two special ensembles *I* and *II*, the success probability can also be pushed up to $\frac{4}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{1-d^2-b^2+2b^2d^2} + \frac{4}{\omega} \times \frac{(b^2-b^4)(d^2-d^4)}{d^2+b^2-2b^2d^2}$ after consuming some extra classical bits.

3 Summary

To summarize, in this paper I have explicitly presented a symmetric and probabilistic scheme for remotely preparing an arbitrary single-qubit state. In the scheme the quantum channel employed by the involved parties consists of two non-maximally entangled two-qubit states. To complete the state preparation, the preparer preforms a two-qubit projective measurement in a properly chosen measurement bases. Then one ministrant is designated to construct the prepared state while another one acts as an assistant. By collaboration, the remote preparation is realized in a probabilistic manner by incorporating an auxiliary qubit and executing appropriate operations including a proper POVM. Furthermore, I have also worked out the success probability, classical communication cost in the scheme, and then explored its applications to two special ensembles of states. It is shown that the RSP can be achieved with higher probability after consuming some extra classical bits while the prepared state is chosen from the two special ensembles. At last, the tripartite RSP scheme is concisely generalized to a multiparty case.

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